Short-Term Memory Capacity in Recurrent Networks via Compressed Sensing

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Summary - Understanding the memory capacity of neural circuits is vital to understanding the performance of tasks such as working memory. In particular, critical questions recently raised in the literature include the effects of network size, connectivity pattern and natural signal statistics on the ability of a neural circuit to encode a temporal memory in its transient dynamics. Recently, results using the tools of compressive sensing (CS) have indicated that the memory capacity of a linear neural circuit could exceed the number of neurons in the network when the temporal signals are sparse (i.e., having few non-zeros) [White et al., 2004, Ganguli and Sompolinsky, 2010].

In this work we perform a full CS analysis of the memory capacity of a neural circuit for general inputs that are sparse in a known basis (a useful model for many natural signals [Olshausen and Field, 1996]). Using a model similar to that proposed in [Ganguli and Sompolinsky, 2010], our results 1) provide signal recovery guarantees for the exact linear network dynamics with no approximations; 2) provide guarantees for *every* temporal sequences sparse in any basis (not average performance), where the memory capacity penalty associated with using a specific basis is well defined; and 3) quantify the dependence of the memory capacity on properties of the neural connectivity pattern.

Our analyses (based on the restricted isometry property [Candès, 2006]) demonstrate that the memory capacity can far exceed the number of neurons in the circuit, and illustrate the magnitude of this excess in two distinct cases: 1) the perfect recovery of finite-length signals that are within the capacity of the circuit, and 2) the recovery error for signals that are beyond the capacity of the circuit (possibly infinitely long), with an exponential forgetting factor in the circuit. These results point to the existence of an optimal signal length (for a given network) where the joint recovery error due to un-recalled signal and recall errors is minimized.

Additional information - Figure 1 shows numerical simulations of memory sequence recovery error from neural circuits.

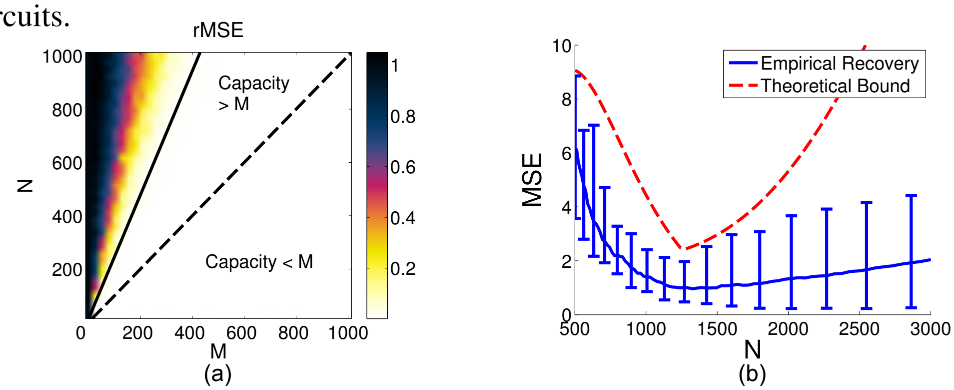

Fig. 1. a) The relative recovery MSE (rMSE) for input stimuli of length N in a network with M nodes when the input is $K = 0.05N$ sparse. Note that the system can accurately recover many signals with length greater than the number of nodes in the network. The solid line indicating rMSE of 0.01 is significantly above the dashed $M = N$ line. b) Recovery MSE for a network with $M = 500$ nodes, an exponential forgetting factor $q = 0.998$, and long (possibly infinite) input sequences. The recovery error for the previous N time steps is a combination of un-recalled signal and recall errors. The theoretical and simulated results show that there is an optimal recovery length for these joint errors (seen as the minimum of the curve). Note that this optimal length is still greater than the number of nodes in the network.

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